

Simple Transmit Diversity for a Convolutionally Coded Multicarrier QAM System

Henrik Schulze

University of Applied Sciences Meschede

Lindenstr. 53, D-59872 Meschede

Tel.: (+49) 291 9910-300, FAX: (+49) 291 9910-40

Email: schulze@fh-meschede.de

Abstract—In this paper, we investigate how a familiar OFDM system with convolutional coding can be improved using Alamouti's two transmit antenna diversity scheme. It turns out that for an ideally interleaved strong convolutional code, only relatively small gains can be achieved. For insufficient interleaving however, the strong code cannot fully exploit its diversity, leading to severe performance degradations. In such situations, two spatially separated transmit antennas may introduce enough additional diversity to compensate this loss.

I. INTRODUCTION

During the last few years, space-time codes (STC) based on transmit antenna diversity have attracted considerable attention. They are especially useful in transmission scenarios where conventional channel coding suffers from a restricted interleaving depth. It is a well-known problem for OFDM systems that insufficient time-frequency interleaving may lead to severe performance degradations, see e.g. [1], [2]. For example, DVB-T and HIPERLAN/2 are communication systems based on OFDM together with convolutional codes that are interleaved only in frequency direction, leading to significant losses for frequency-flat fading channels.

In this paper, we investigate how this problem can be overcome by using space-time coding. To keep the changes to well-designed existing systems as small as possible, we maintain the usual convolutionally coded QAM modulation and combine it with an inner space-time code. Alamouti's simple 2-antenna transmit diversity scheme [3] seems to be ideally suited to fulfill this requirement. This STC scheme has no coding gain, but the full diversity degree of order two. Our *Ansatz* is to get all the desired coding gain from the outer convolutional code and use the inner STC only to get some additional diversity for the case of insufficient interleaving where the outer code cannot exploit its full diversity degree.

The paper is organized as follows: In Section II, we describe the system setup. In Section III, we discuss

the geometrical interpretation of the Alamouti scheme which provides a visual understanding of the receiver structure and the performance. In Section IV, we discuss outer coding. In Section V we show numerical simulations, and in Section VI, we draw some conclusions.

II. SYSTEM SETUP

The system setup is shown in Fig. 1. The data bits

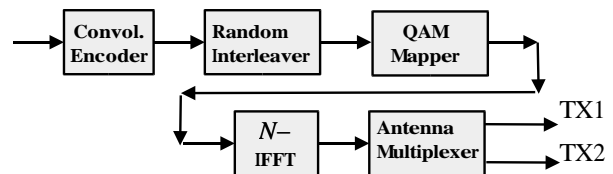


Fig. 1. System Setup Block Diagram

will be encoded by a standard convolutional encoder of constraint length 7 with generators $(133, 171)_{oct}$. Optionally, higher code rates can be obtained by puncturing. The coded bitstream will be interleaved by a block interleaver that performs a random interleaving over $\log_2(M)I$ bits, corresponding to I/K OFDM symbols, where K is the number of OFDM subcarriers. The block of interleaved bits will be mapped on $I M$ -QAM symbols with conventional Gray mapping. The complex QAM symbols will be processed block-wise by an IFFT device of FFT length N ($N \geq K$), resulting into a stream of complex time domain samples that are grouped into vectors $x_i[n]$, $n = 0, 1, \dots, N - 1$ of length N , each corresponding to one OFDM symbol numbered by i . These vectors will be grouped into pairs, and each pair will be processed by the transmit diversity multiplexer in following way: For the first time slot, the first OFDM symbol vector $x_1[n]$ will be sent to transmit antenna 1 (TX1), and the second vector $x_2[n]$ will be sent to transmit antenna 2 (TX2). At the second time slot, the time reversed and complex conjugate second vector $x_2^*[N - n]$ will be sent to TX1, and $-x_1^*[N - n]$ will be sent to TX2. After this STC procedure, that may

more appropriately be regarded as an *antenna multiplexing*, the guard interval will be inserted separately to the resulting OFDM symbols for the respective antennas.

We note that if we switch off TX2, except for the complex conjugate in the second symbol, our setup is just a standard transmission setup like it is used in the standards of DVB-T, HIPERLAN/2, and DRM (with some modifications concerning the QAM modulation).

III. TRANSMIT DIVERSITY FOR TWO ANTENNAS

A. The Transmission Scheme

Let us denote the respective discrete Fourier transforms of $x_1[n]$ and $x_2[n]$ by $s_1[k]$ and $s_2[k]$, $k = 0, 1, \dots, N - 1$. We write each pair of QAM symbols at frequency index k as a column vector $\mathbf{s}[k] = (s_1[k], s_2[k])^T$. Using well known properties of the discrete Fourier transform, we get a transmission scheme for each pair of QAM symbols like shown in Table I.

TABLE I
TX DIVERSITY MULTIPLEXER (FREQUENCY DOMAIN)

Antenna	TX1	TX2
time slot 1	$s_1[k]$	$s_2[k]$
time slot 2	$s_2^*[k]$	$-s_1^*[k]$

For a given subcarrier index k , this is just Alamouti's [3] well known transmit diversity scheme. We denote the complex fading amplitudes at frequency k for antenna 1 and 2 a by $c_1[k]$ and $c_2[k]$, respectively. We assume that the channel does not vary during the two time slots. We further assume that all ISI has been absorbed by the guard interval, so that a time-discrete channel model is justified for each frequency index k . Let $r_1[k]$ be the received symbol at time slot 1 and $r_2[k]$ the *complex conjugate* received symbol at time slot 2 and write them as a column vector $\mathbf{r}[k] = (r_1[k], r_2[k])^T$. The time-discrete channel is then given by

$$\mathbf{r}[k] = \mathbf{C}[k]\mathbf{s}[k] + \mathbf{n}[k], \quad (1)$$

where $\mathbf{n}[k]$ is the Gaussian noise vector with variance $\sigma^2 = N_0/2$ in each real dimension and

$$\mathbf{C}[k] = \begin{pmatrix} c_1[k] & c_2[k] \\ -c_2^*[k] & c_1^*[k] \end{pmatrix}. \quad (2)$$

is the *channel matrix* at frequency k . In the following discussion, we will drop the frequency index k .

B. Geometrical Interpretation

We observe that the channel matrix \mathbf{C} has the property

$$\mathbf{C}^\dagger \mathbf{C} = \mathbf{C} \mathbf{C}^\dagger = c^2 \cdot \mathbf{I}, \quad (3)$$

where \mathbf{C}^\dagger is the Hermitian conjugate (=transposed and complex conjugate) of the matrix \mathbf{C} , \mathbf{I} is the identity matrix, and

$$c^2 = |c_1|^2 + |c_2|^2 \quad (4)$$

is the total power of the composed fading channel belonging to two transmit antennas. Equation (3) means that the channel matrix \mathbf{C} can be written as $\mathbf{C} = c\mathbf{U}$, where \mathbf{U} is a unitary matrix. Unitary matrices (like orthogonal matrices for real vector spaces) leave Euclidean distances invariant. They can be visualized as rotations (maybe combined with reflections) in a Euclidean space. This means that the transmission channel given by Eq. (1) can be separated into three parts:

1. A rotation in 2 complex (or 4 real) dimensions.
2. An attenuation by the composed fading amplitude c .
3. An AWGN channel.

Keeping in mind that multiplicative fading is just a phase rotation together with an attenuation by a real fading amplitude, we can now interpret this transmission as a generalization of the familiar multiplicative fading from one to two complex dimensions: A generalized phasor \mathbf{s} will be rotated by \mathbf{U} and attenuated by the fading amplitude $c = \sqrt{|c_1|^2 + |c_2|^2}$. Since a rotation does not influence the performance, it is only affected by this fading amplitude. Following this argument, it can easily be seen that the performance analysis (i.e. of the error event probabilities given by Euclidean distances) is completely equivalent to that for one transmit and 2 receive antennas.

C. Diversity Combining at the Receiver

For a time discrete transmission given by $\mathbf{r} = \mathbf{C}\mathbf{s} + \mathbf{n}$, the optimum receiver has to evaluate the squared Euclidean distance $\|\mathbf{r} - \mathbf{C}\mathbf{s}\|^2$. It follows from the Maximum Likelihood principle that the most probable transmitted symbol vector \mathbf{s} is the one that minimized this squared Euclidean distance, where the minimum has to be taken over the set of all possible transmit vectors \mathbf{s} . With additional outer coding, we need to pass soft decision values to the outer decoder. These are given by *log-likelihood ratios*. Each transmit vector \mathbf{s} corresponds to a vector $\mathbf{b} = (b_1, \dots, b_m)$ of binary digits. The conditioned log-likelihood ratio (LLR) of bit b_i number i for

the received vector \mathbf{r} is given by

$$\Lambda(b_i|\mathbf{r}) = \log \left(\frac{\sum_{\mathbf{s} \in \mathcal{S}_i^{(0)}} \exp \left(-\frac{1}{2\sigma^2} \|\mathbf{r} - \mathbf{C}\mathbf{s}\|^2 \right)}{\sum_{\mathbf{s} \in \mathcal{S}_i^{(1)}} \exp \left(-\frac{1}{2\sigma^2} \|\mathbf{r} - \mathbf{C}\mathbf{s}\|^2 \right)} \right) \quad (5)$$

$\mathcal{S}_i^{(0)}$ is the set of transmitted signals \mathbf{s} with $b_i = 0$ and $\mathcal{S}_i^{(1)}$ is the set of transmitted signals \mathbf{s} with $b_i = 1$.

For the transmit diversity scheme under consideration, the inverse channel matrix exists and is given by $\mathbf{C}^{-1} = c^{-1}\mathbf{U}^{-1}$. Using eq. (3), we can write the squared Euclidean distance as

$$\|\mathbf{r} - \mathbf{C}\mathbf{s}\|^2 = c^2 \|\mathbf{C}^{-1}\mathbf{r} - \mathbf{s}\|^2. \quad (6)$$

$\mathbf{C}^{-1}\mathbf{r}$ can be interpreted as a back rotation \mathbf{U}^{-1} of the receive vector, followed by a rescaling (or equalization) with the inverse fading amplitude c^{-1} . Each decision has then be weighted by the the channel power c^2 to pass the channel reliability information to the outer decoder. Obviously, this receiver just generalizes the multiplicative fading channel QAM receiver from two to four real dimensions.

Alternatively, and especially useful for PSK transmission, one may write

$$-\|\mathbf{r} - \mathbf{C}\mathbf{s}\|^2 = 2\Re\{\langle \mathbf{C}^\dagger\mathbf{r}|\mathbf{s} \rangle\} - c^2\|\mathbf{s}\|^2 + const., \quad (7)$$

where $\langle .|. \rangle$ denotes the scalar product in the (complex) vector space of signals. $\mathbf{C}^\dagger\mathbf{r}$ is a linear operation on the received signal that can be interpreted as a generalization of the well-know maximum ratio combiner (MRC). The scalar product can be interpreted as a projection on (or cross- correlation with) the transmitted signal \mathbf{s} . The second term $c^2\|\mathbf{s}\|^2$ is the transmit signal energy. It can be ignored for PSK transmission.

IV. TRANSMIT DIVERSITY WITH OUTER CODING

The transmit antenna diversity scheme does not require additional bandwidth and shows already a significant gain over uncoded transmission. However, in fading channels usually strong channel coding is applied.

The additional diversity gain for a coded transmission is easy to evaluate because the transmission system is completely equivalent to a convolutional code with an outer repetition code which can easily be analysed by well-known methods.

The bit error probability of a convolutional code is tightly bounded by the union bound

$$P_b \leq \sum_{d=d_{free}}^{\infty} c_d P_d. \quad (8)$$

d_{free} is the free distance of the code, the error coefficients c_d can be found in tables (see e.g. [8]), and P_d is the error probability for d -fold diversity. For independent Rayleigh fading and 4-QAM (QPSK), analytical expressions for P_d can be found in [6], [7]. For higher level QAM modulation (bit interleaved coded modulation BICM), the methods described in [4] and [5] can be applied to obtain tight upper bounds for P_d . It is clear from the analysis of the error probability of convolutional codes that 2-fold diversity (= a repetition code) after coding and interleaving just doubles the distance for each error event. This means that we have only have to replace have P_d by P_{2d} in the above equation.

V. SIMULATIONS

A. Channel Simulation Parameters

We denote the OFDM symbol duration by $T_S = T + \Delta$, where T the is useful OFDM symbol duration, and $\Delta = T/4$ is the guard interval. We simulate a time and frequency selective Rayleigh fading channel with an isotropic Doppler spectrum with maximum shift $\pm f_{Dmax}$ and a rectangular delay power spectrum characterized by the delay spread τ_m .

The time variance of the channel is characterized by $f_{Dmax}T_S$, the frequency variance by $\tau_m B$, where $B = K/T$ is the transmission bandwidth. For a fixed ratio τ_m/Δ , and because $T = 4\Delta$, the number of carriers K can directly be regarded as a measure for the frequency selectivity.

Fast fading (high $f_{Dmax}T_S$) may cause degradations because the complex channel amplitudes vary to much over two time slots. On the other hand, slow (small $f_{Dmax}T_S$) and flat (small K) fading may cause insufficient interleaving.

In our simulations, we always assume ideal channel estimation. We consider a channel with moderate (but not extremely small) delay spread $\tau_m = 0.15\Delta$ compared to the guard interval. Our simulations assume a moderately fast fading channel with $f_{Dmax}T_S = 0.01$ for which coded QAM systems are known to work well, see [1], [2].

B. Simulation Results

Figure 2 shows the bits error rate (BER) as a function of E_b/N_0 for a such a channel and 16-QAM transmission with code rate $R_c = 1/2$ and a huge time and frequency interleaving over the bandwidth of $K = 768$ carriers and the time of $I/K = 360$ OFDM symbols. The curves fit well to the union bounds for the ideally interleaved Rayleigh channel. The gain due to antenna di-

versity lies between 1 and 2 dB in the region of interest, which is much smaller than in the uncoded case, where gains of about 15 dB are obtained at $BER = 10^{-4}$, see [3].

Figure 3 shows the performance curve for the same channel with a small relative bandwidth ($K = 48$) and no time interleaving. This system, which can be compared with a HIPERLAN/2 system, shows severe degradations of 6.5 dB at $BER = 10^{-5}$ due to insufficient interleaving if no diversity has been applied. We conclude that such a low number of carriers means that the OFDM system is a narrowband system compared to the channel. Figure 3 shows that only one additional transmit antenna can compensate approx. 4 dB of that narrowband loss. If we increase the bandwidth by a factor of 16 to $K = 768$, similar simulations show that the interleaving only in frequency direction means only a loss of 0.8 dB for one TX antenna and 0.4 dB for 2 TX antennas. We may call this a wideband system. However, smaller values of τ_m will cause severe degradations [1], [2] and antenna diversity would be very helpful in such situations.

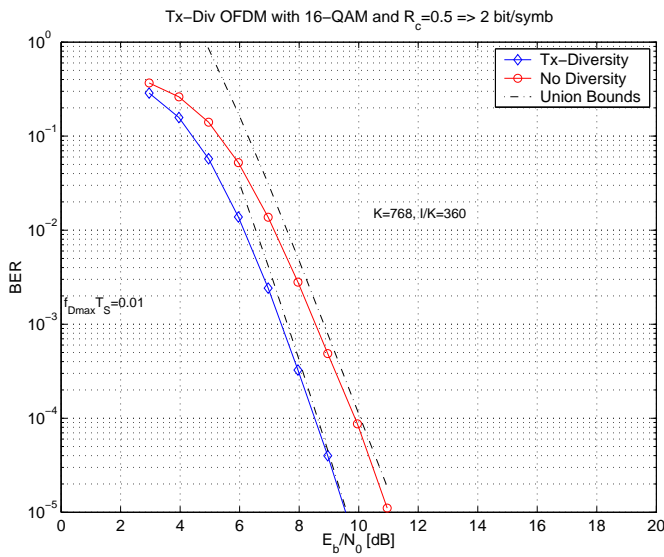


Fig. 2. Bit error rates for the coded system with and without diversity for $f_{Dmax}T_S = 0.01$ and $\tau_m/\Delta = 0.15$ (huge interleaver)

VI. DISCUSSION AND CONCLUSIONS

We have seen that Alamouti's transmit diversity can be visualized as generalized multiplicative fading in a four-dimensional signal space. The fading statistics is equivalent to that of conventional 2-fold diversity and can be analysed analytically by standard methods. It turns out that for an ideally interleaved strong convo-

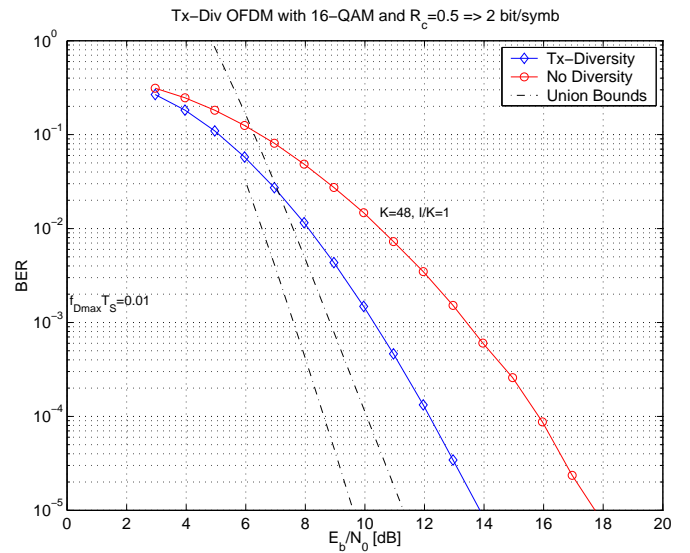


Fig. 3. Bit error rates for the coded system with and without diversity for $f_{Dmax}T_S = 0.01$ and $\tau_m/\Delta = 0.15$ (small interleaver)

lutional code, one additional transmit antenna provides only relatively small gains of 1-2 dB in E_b/N_0 . For scenarios with insufficient interleaving however, this second antenna may compensate a big part of the loss due to poor interleaving.

REFERENCES

- [1] H. Schulze, "Möglichkeiten und Grenzen des Mobilempfang von DVB-T", 3. OFDM-Fachgespräch in Braunschweig, 29.-30. Sep. 1998
- [2] H. Schulze, "The Performance of a Coded Multicarrier 64-QAM System with Channel Estimation", 4th Int. OFDM Workshop in Hamburg, pp. 15.1-15.4, Sep. 1999
- [3] S. M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications", *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.
- [4] G. Caire, G. Taricco, E. Biglieri. Bit-Interleaved Coded Modulation. *IEEE Transactions on Information Theory*, Vol. IT-44(3), pp. 927-946, May 1998.
- [5] H. Schulze. System Design for Bit-Interleaved Coded Square QAm in a Rician Fading Channel. To appear in *European Transactions on Telecommunications*.
- [6] J. Hagenauer. Fehlerkorrektur und Diversity in Fading-Kanälen. *AEÜ* Vol. 36, pp. 337-344, 1982.
- [7] A. Proakis. *Digital Communications*, 3rd ed., McGraw-Hill, New York, 1995.
- [8] J. Hagenauer. Rate Compatible Punctured Convolutional Codes (RCPC Codes) and their Applications. *IEEE Transactions on Communications*, Vol. COM-36, pp. 389-400, 1988.