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The Performance of Multicarrier CDMA for the Correlated Rayleigh Fading Channel

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Abstract **We present an new method for the analytical calculation of the error probability in a Rayleigh fading channel with correlated fading amplitudes. Even though the method can be applied for more general problems, we restrict ourselves on the investigation of a multicarrier CDMA transmission. The frequency correlation between the fading amplitudes of the carriers is given by the delay power spectrum of the channel. We generalize the well-known analytical expressions for the pairwise error probability of the uncorrelated Rayleigh channel to this case of correlated fading. This probability can be expressed in a closed form by an integral over a finite interval which can be easily evaluated numerically. We present some examples to show how much of the diversity of a code remains useful if the transmission channel does not provide an arbitrary degree of diversity.**

Keywords **Multicarrier CDMA, Diversity, Coding, Correlated Rayleigh fading**

1. Introduction

Reliable transmission over a mobile Rayleigh fading channel needs to make use of some statistical independence that must be provided by the channel. The simplest method is to receive the same symbol over different antennas, different frequencies, different time slots, or different echo paths (for a RAKE receiver). This familiar form of diversity (we will call it repetition diversity because it is nothing but a repetition code) leads to a power law $P_b \propto (E_b/N_0)^{-L}$ for L *independent* diversity branches. The same formula for L-fold repetition diversity with maximum ratio combining (MRC) applies for the pairwise error probability for maximum likelihood (ML) decoding of an error-correcting code, if L is the Hamming distance between the two code words and ideal interleaving can be assumed. Analytical formulas for this error probabilities in a Rayleigh fading channel with independent fading amplitudes are well known [1, 2]. However, for many relevant mobile transmission scenarios, the assumption of independent fading is too optimistic. For example, the interleaving depth may be restricted by the allowed decoding delay or the spreading bandwidth is not very large compared to the correlation bandwidth.

In this paper, we present a new method to calculate this error probability for the case that the L diversity branches are *correlated*. First, we make use of the fact that the correlated fading process is unitary equivalent to an uncorrelated fading process. Practically this means that we have to solve the eigenvalue problem of the autocorrelation matrix. Then we make use of the alternative form of the Gaussian probability integral for which the averaging over independent fading amplitudes of different power can easily be performed. To obtain the error rate, one has then to compute a finite integral numerically.

An interesting application where the question of channel diversity becomes important is CDMA transmission. Spreading the signal over a larger bandwidth provides the receiver with more uncorrelated information only if the transmission bandwidth becomes significantly larger than the correlation bandwidth of the channel. Otherwise there is a loss due to residual correlations. In the time domain, e.g. for DS-CDMA with a RAKE receiver, the restricted bandwidth leads to correlations between the echo paths (RAKE fingers). In this paper, we restrict ourselves on the frequency domain analysis and consider a multicarrier (MC) CDMA [3, 4] system. In such a system, typically an inner repetition (RP) code or a Walsh-Hadamard (WH) code is used for spreading. Optionally, an outer convolutional code can be used. The performance of different channel coding schemes for MC-CDMA in a Rayleigh fading channel with independent fading amplitudes has been studied analytically in [5]. We carry out a similar performance analysis only for the inner (RP or WH) coding and investigate the degradations due to the correlations in a Rayleigh fading channel. We restrict ourselves on the simple model that the multiuser interference (MUI) can be treated like an additional noise term. In this case it is sufficient to analyse only the performance of a single user (MC *spread spectrum*) system in a Gaussian noise channel. The MUI can then easily be included like shown e.g. in [5].

This paper is organized as follows: We describe the transmission system in Section 2. In Section 3, we derive the formula to calculate the error probability for correlated fading. In Section 4, we discuss the concept of diversity for a correlated fading channel. We then evaluate our formula numerically for the MC-CDMA system in Section 5. We discuss other applications in Section 6 and draw some conclusions in Section 7.

2. System model

We consider a multicarrier CDMA system with BPSK modulation [3–5]. One way to introduce spreading is

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to transmit a pseudo-noise (PN) sequence with plus or minus sign for each data bit. In fact, this corresponds to a simple repetition code and BPSK modulation followed by multiplication with the PN sequence for user separation. Another way is to use low-rate Walsh-Hadamard (WH) codes with BPSK followed by multiplication with the PN sequence. As it will be explained later, it may be advantageous to apply a pseudo random permutation to the WH code word that we regard as an *intra code word interleaver*. Let K be the number of sub-carriers. We assume that an integer number of code words of length N will be transmitted in one time slot, i.e. K is an integer multiple of N. We assume that the channel is frequency selective, but not time selective (slowly varying). Intersymbol interference is assumed to be absorbed e.g. by a guard interval. We may now work with a frequency discrete channel model.

For a fair comparison between different coding schemes, it is a good practice to regard error rates as as function of E_b/N_0, where E_b is the energy per data bit and N_0 is the one-sided white Gaussian noise density. For BPSK and a code of rate R_c we have

$$\frac{E_S}{N_0} = R_c \frac{E_b}{N_0}, \tag{1}$$

where E_S is the energy per BPSK symbol (i.e. the *chirp energy*). Note that the energy loss due to the OFDM guard interval is not included here. For an L-fold repetition code, we have $R_c = 1/L$, for a WH(M, $\log_2 M$, $M/2$) code, we have $R_c = \log_2 M/M$.

A code word will be mapped on a sequence of BPSK symbols $x_i \in \{\pm\sqrt{E_S}\}$, $i \in \{1, ..., N\}$. The vector $\mathbf{x} = (x_1, ..., x_N)^{\mathrm{T}}$ corresponding to this code word will be transmitted over the discrete channel given by

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{n}. \tag{2}$$

$\mathbf{y}$ is the vector of received symbols and $\mathbf{n}$ is the complex Gaussian noise vector with variance $\sigma^2 = N_0/2$ in each real component. The fading is described by the diagonal matrix $\mathbf{C}$. The diagonal is the vector of fading amplitudes $(c_1, ..., c_N)^{\mathrm{T}}$. All vectors indices should be understood as frequency indices. The fading amplitudes are normalized to average power one. We assume Rayleigh fading and the WSSUS (wide-sense stationary uncorrelated scattering) model, see e.g. [2]. This means that the fading is a complex-valued Gaussian process with mean zero, and the 2-D autocorrelation depends only on time and frequency differences. In this model, the statistics of the fading amplitudes c_i sampled at frequencies f_i is completely determinded by the elements

$$\mathrm{E}\{c_i c_k^*\} = K(f_i - f_k). \tag{3}$$

of the (Hermitian) autocorrelation matrix. $K(f)$ denotes the frequency autocorrelation function of the WSSUS model.

3. Analytical evaluation of the pairwise error probability

All code words are assumed to be sent with the same *a priori* probability. Then, for optimum (ML or MRC) detection, the pairwise error probability that a vector $\mathbf{x}$ will be send but another vector $\mathbf{x}'$ will be detected under the condition of a fixed channel $\mathbf{C}$ is given by

$$P(\mathbf{x} \to \mathbf{x}'|\mathbf{C}) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{|\mathbf{C}\mathbf{x} - \mathbf{C}\mathbf{x}'|^2}{8\sigma^2}}\right). \tag{4}$$

If the two code words differ at exactly $L \leq N$ positions $i_1, ..., i_L$ this error probability equals

$$P(\mathbf{x} \to \mathbf{x}'|\mathbf{C}) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\sum_{l=1}^{L} |c_{i_l}|^2 \frac{E_S}{N_0}}\right). \tag{5}$$

The pairwise error probability $P_L(\mathbf{x} \to \mathbf{x}')$ for two code word that differ at L positions will be obtained by averaging over the fading amplitudes. If they are independent and identically Rayleigh distributed, the sum of squared amplitudes can be shown to be χ^2 distributed and the average can be performed by evaluating one integral, leading to the well known formula for diversity with L independent branches, see [1, 2]:

$$P_L(\mathbf{x} \to \mathbf{x}') = \frac{1}{2}\left[1 - \mu \sum_{l=0}^{L-1} \binom{2l}{l} \left(\frac{1-\mu^2}{4}\right)^l\right] \tag{6}$$

with

$$\mu = \sqrt{\frac{E_S/N_0}{1 + E_S/N_0}}. \tag{7}$$

Fig. 1 shows the bit error probabilities $P_b = P_L(\mathbf{x} \to \mathbf{x}')$ for the L-fold repetition diversity obtained from eq. (6) for $L = 1, 2, 4, 8, 16, 32, 64$ in comparison with the error rate for uncoded transmission in the AWGN channel. It is obvious from the figure (and will be shown below) that for $L \to \infty$, the diversity curves converge to the AWGN curve.

Recently, it has been pointed out by Simon and Divsalar [6] that the (solved) problem of calculating the error rate for diversity with L independent branches as well as some other (unsolved) problems can be treated in a very elegant way by using the alternative representation of the Gaussian probability integral given by

$$\frac{1}{2}\,\mathrm{erfc}(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{\sin^2\theta}\right) d\theta. \tag{8}$$

We shall give a simple proof in the Appendix. With this formula, the averaging over the L Rayleigh distributed

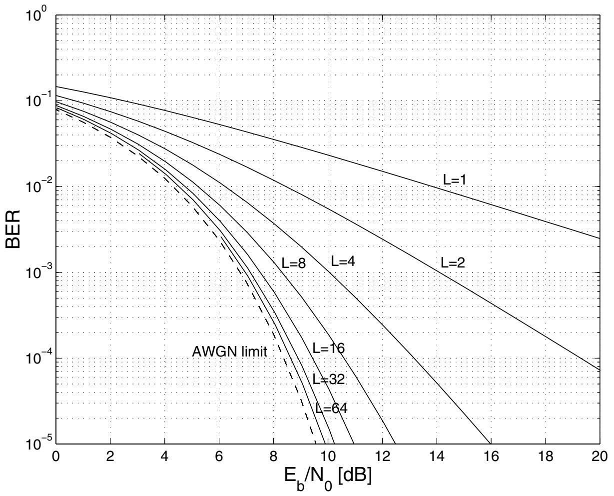

Fig. 1. Bit error probabilities (BER) for the L-fold repetition diversity obtained from eq. (6) for $L = 1, 2, 4, 8, 16, 32, 64$.

fading amplitudes $c_{i_1}, ..., c_{i_L}$ in equation (5) can be easily performed for independent Rayleigh fading since the exponential factorizes, leading to the expression:

$$P_L(\mathbf{x} \to \mathbf{x}') = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{1}{1 + \frac{1}{\sin^2\theta} \frac{E_S}{N_0}} \right)^L d\theta. \quad (9)$$

This integral over a finite interval can be solved using the residue theorem. For practical purposes, it is also very easy to evaluate it numerically.

We can now use this alternative representation of the probability integral to treat also the case of correlated Rayleigh fading. We note that since the autocorrelation matrix of the fading is Hermitian, it can be transformed to a diagonal one by a unitary matrix $\mathbf{U}$. This is just the well known Karhunen-Loeve transform, see e.g. [7, 8]. By this unitary transform, a channel given by a vector $\mathbf{c}_L = (c_{i_1}, ..., c_{i_L})^{\mathrm{T}}$ will be mapped to a vector $\mathbf{b}_L = (b_1, ..., b_L)^{\mathrm{T}}$ of uncorrelated fading amplitudes

$$\mathrm{E}\{b_i b_k^*\} = \lambda_i \delta_{ik} \quad (10)$$

according to

$$\mathbf{b}_L = \mathbf{U}^{-1} \mathbf{c}_L. \quad (11)$$

The real numbers λ_i are just the eigenvalues of the complex autocorrelation matrix of the fading. Without loss of generality, we assume ordering $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_L \geq 0$. Note that the unitary matrix $\mathbf{U}$ and therefore also the eigenvalues λ_i depend on the L positions $i_1, ..., i_L$. Since the transform is unitary, it follows that

$$\sum_{i=1}^{L} |b_i|^2 = \sum_{l=1}^{L} |c_{i_l}|^2 \quad (12)$$

and therefore

$$\sum_{i=1}^{L} \lambda_i = \mathrm{E}\Big\{ \sum_{i=1}^{L} |b_i|^2 \Big\} = \mathrm{E}\Big\{ \sum_{l=1}^{L} |c_{i_l}|^2 \Big\} = L. \quad (13)$$

We can interpret λ_i as the power of the i-th diversity branch of the *equivalent independently fading channel* obtained by a unitary transform.

The averaging over L in equation (5) can now easily be performed, since the exponential factorizes, leading to the expression:

$$P_L(\mathbf{x} \to \mathbf{x}') = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^{L} \frac{1}{1 + \frac{\lambda_i}{\sin^2\theta} \frac{E_S}{N_0}} d\theta. \quad (14)$$

Again, this integral can be evaluated numerically.

4. The concepts of diversity and coding gain

Eq. (14) for the pair error probability for two code words with Hamming distance L can be written as a function of E_b/N_0 in the following way:

$$P_L(\mathbf{x} \to \mathbf{x}') = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^{L} \frac{1}{1 + \frac{1}{L} R_c L \frac{\lambda_i}{\sin^2\theta} \frac{E_b}{N_0}} d\theta. \quad (15)$$

We note that an asymptotic decay proportional to $(E_b/N_0)^{-L}$ is only given if $\lambda_i > 0$ for all eigenvalues. Furthermore, many or most of the eigenvalues may be so small that they are of no relevance at reasonable signal-to-noise ratios. Therefore, the decay will be weaker in most cases. The *diversity branch spectrum* $\{\lambda_i\}_{i=1}^{L}$ of the *equivalent independently fading channel* has to be evaluated for each correlated fading channel that is given by the autocorrelation function. It is obvious that a Hamming distance of the code that is higher than the relevant number of eigenvalues does not significantly influence the steepness of the error curves. We may speak of the diversity degree of the code (more precisely: of the error event) given by the Hamming distance and of the diversity degree of the channel given by the number of relevant eigenvalues.

However, if the channel does not provide the full *diversity* gain, Eq. (15) shows that there may still be a *coding gain* of

$$G_L(\mathbf{x} \to \mathbf{x}') = 10 \lg(R_c L)\, \mathrm{dB} \quad (16)$$

if $R_c > 1/L$. This is just the gain compared to the simple repetition code (that has no coding gain) and corresponds to the coding gain for the AWGN channel. Therefore we have two things to consider: The diversity of the code and the channel that is responsible for the steepness of the error curves, and the coding gain that shifts the curves to the left. Note that if this shift is big enough, the coded system may need a lower E_b/N_0 at a given bit error rate in the Rayleigh channel than the uncoded in the AWGN channel. The number of neighbors and the number of different data bits (error coefficients) are also relevant, but this requires a detailed analysis of the structure of the code. For our numerical analysis, we restrict ourselves on codes with a very simple structure.

5. Numerical analysis

5.1 Repetition diversity

For the L-fold repetition code, there are only two code words and the pairwise error probability equals the bit error probability P_b given by

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L}\frac{1}{1+\frac{1}{L}\frac{\lambda_i}{\sin^2\theta}\frac{E_b}{N_0}}d\theta. \tag{17}$$

We first note that it can be seen very easily from this formula that for identical, uncorrelated fading amplitudes ($\lambda_i = 1$ for all i) the AWGN channel will be reached in the limit $L \to \infty$ since $(1+x/L)^L \to e^x$ for all x and the limit can be performed under the integral.

The decay depends on the diversity branch spectrum $\{\lambda_i\}_{i=1}^{L}$ of the eigenvalues. For the numerical investigations, we will assume an exponential delay power spectrum

$$S_D(\tau) = \frac{1}{\tau_m}e^{-\tau/\tau_m}\epsilon(\tau), \tag{18}$$

where τ_m is the mean delay and $\epsilon(\tau)$ is the Heaviside function. The corresponding frequency autocorrelation function is given by

$$K(f) = \frac{1}{1+j2\pi f\tau_m}. \tag{19}$$

Fig. 2 shows the first 16 eigenvalues for a spreading factor $L = 64$ and different values of the bandwidth B. We define a *normalized* bandwidth $X = B\tau_m$. We have assumed that the BPSK symbols are equally frequency-spaced over the bandwidth. We see that for a small bandwidth (e.g. $X = 1$ corresponding to 1 MHz for $\tau_m = 1$ µs), the equivalent independent fading channel has only a low number of diversity branches of significant power. We found that the *diversity branch spectrum* like shown in Fig. 2 is nearly independent of L if L is significantly greater than X. It is therefore a very useful quantity to characterize the diversity that can be provided by the channel.

A look at the eigenvalues gives a first glance how many diversity branches of the equivalent independent fading channel contribute significantly to the transmission. It finds it's reflection in the performance curves. Fig. 3 shows the pairwise (= bit) error probability for $L =$ 32 and $X = B\tau_m = 0.5, 1, 2, 4, 8, 16$. The high diversity degree of the repetition code ($L = 32$) can show a high diversity gain if the equivalent channel has enough independent diversity branches of significant power. This is the case for e.g. $X = 16$, but not for $X = 1$ or $X = 2$. For low X a lower repetition rate L would have been sufficient.

Fig. 4 shows the bit error probability for $L = 10$ and the same values of X. For low X, the curves of Fig. 3 and Fig. 4 are nearly identical. For higher X, the curves of Fig. 4 run into a saturation that is given by the performance curve of the independent Rayleigh fading. For $X = 8$, this limit has been practically achieved. There is still a gap of nearly 2 dB to the AWGN limit at the bit error rate of 10^{-4}. We will demonstrate now that using codes with a coding gain $G > 0$ dB like WH codes we will get beyond this AWGN curve.

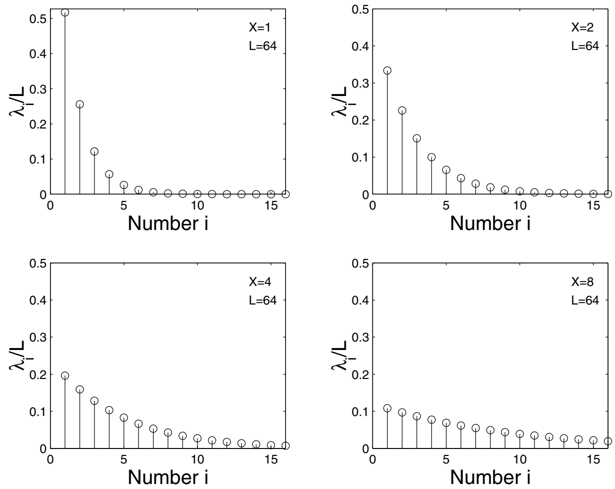

Fig. 2. Diversity branches of the equivalent independent fading channel and normalized bandwidth $X = B\tau_m = 1, 2, 4, 8$.

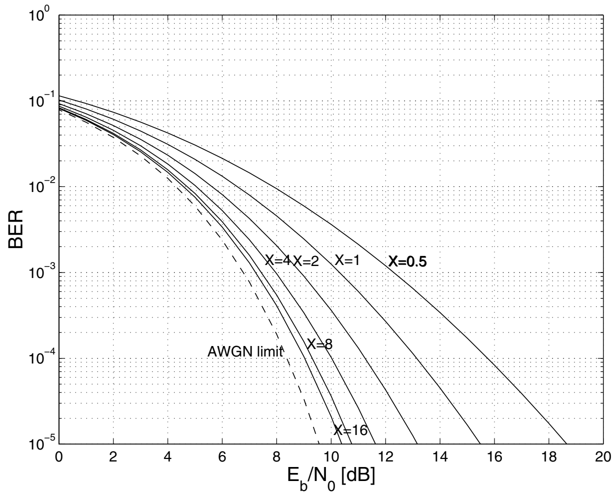

Fig. 3. Bit error probabilities for 32-fold repetition diversity with $X = 0.5, 1, 2, 4, 8, 16$.

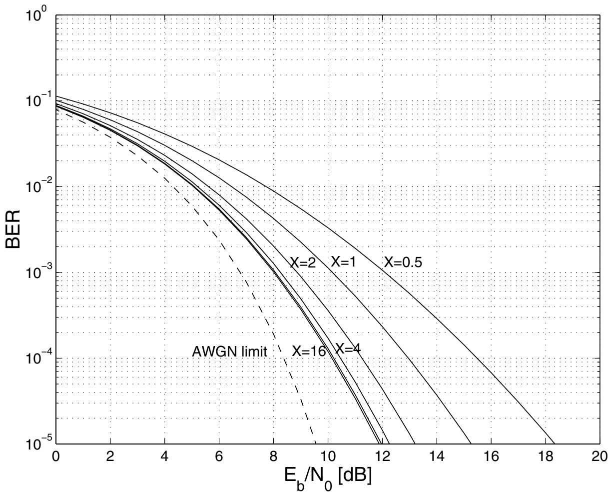

Fig. 4. Bit error probabilities for 10-fold repetition diversity with $X = 0.5, 1, 2, 4, 8, 16$.

5.2 Walsh-Hadamard codes

We now consider a WH(M, $\log_2 M$, $M/2$) code of M different code words, code rate $R_c = \log_2 M/M$, and constant weight $M/2$, which means that each pair of code words differs in $L = M/2$ symbol positions. The pairwise error probability

$$P_{M/2}(\mathbf{x} \to \mathbf{x}') = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^{M/2} \frac{1}{1 + \frac{\log_2 M}{M} \frac{\lambda_i}{\sin^2\theta} \frac{E_b}{N_0}} d\theta. \quad (20)$$

is different for different pairs $\mathbf{x}$ and $\mathbf{x}'$ and depends on the L positions $i_1, ..., i_L$ where the two code words differ. The autocorrelation matrix and therefore the eigenvalues λ_i depend on these positions. The correlations of the channel sampled at these positions can be very different. Assume, for example, that the all-zero code word has been send, which corresponds to the vector $\mathbf{x_1}$ with the symbol $+\sqrt{E_S}$ in each position. If $\mathbf{x}'$ is the vector with symbol $+\sqrt{E_S}$ in position $i = 1, 2, .., M/2$ and $-\sqrt{E_S}$ in position $i = M/2+1, ..., M$, there is much more correlation of the fading amplitudes than for $\mathbf{x}'$ being the vector with alternating sign (i.e. $x_i' = (-1)^{i-1}\sqrt{E_S}$). Numerical evaluation of the pairwise error probabilities shows that the curves for different pairs can differ by several dezibels. We found that this effect – that is unknown for uncorrelated fading – can be mitigated significantly if we introduce an *intra code word interleaving* which is just a pseudo random permutation of the coded symbols.

For practical considerations, the bit error probability P_b is much more interesting than the pairwise error probability. The union bound for detecting a wrong code word (block error probability) is given by

$$P_{Block} \leq \sum_{m=2}^{M} P_{M/2}(\mathbf{x_1} \to \mathbf{x_m}), \quad (21)$$

where we have assumed that the code word $\mathbf{x_1}$ has been send. For the bit error probability it follows the union bound

$$P_b \leq \frac{M/2}{M-1} \sum_{m=2}^{M} P_{M/2}(\mathbf{x_1} \to \mathbf{x_m}). \quad (22)$$

In Fig. 5, we have evaluated this union bound of the bit error rate for independent, identically distributed Rayleigh-fading and $M = 2, 4, 8, 16, 32, 64$. This serves as a reference to compare with correlated fading.

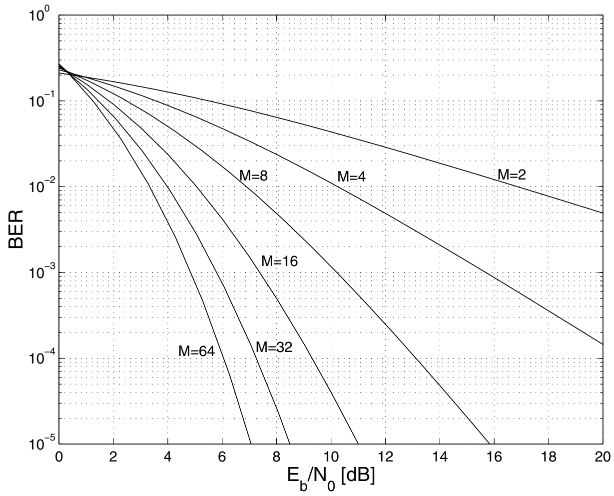

Fig. 5. Bit error rates (Union Bounds) for uncorrelated Rayleigh fading and the WH codes with $M = 2, 4, 8, 16, 32, 64$.

Fig. 6 shows the union bound for the bit error rate for $M = 64$ and different values of the normalized bandwidth X. The WH(64,6,32) code has nearly the same spreading factor R_c^{-1} like the $L = 10$ repetition code, but the performance is much better. One reason is the high diversity degree that is given by the Hamming distance $d_H = 32$. The other reason is that we have a coding gain $G = \lg(d_H R_c)$ dB > 0 dB. Therefore, this code even outperforms the $L = 32$ repetition code with the same Hamming distance. For high values of X, the performance is significantly better than the AWGN limit.

Fig. 7 shows the union bound for the bit error rate for $M = 32$ and different values of the normalized bandwidth X.

From Figs. 6 and 7 we conclude that a high Hamming distance of a code only leads to a steep decay of the performance curves if the equivalent channel has enough independent diversity branches, i.e. X must be large enough. For small X, there are not much differences in the performance of both WH codes, but there is a very significant dependence on X. For the strong WH code with $M = 64$, Fig. 6 shows that there is a loss of approx. 10 dB (at $BER = 10^{-4}$) between the most frequency selective channel with $X = 16$ and the very flat one with $X = 0.5$. Since a system in a mobile environment must operate in very

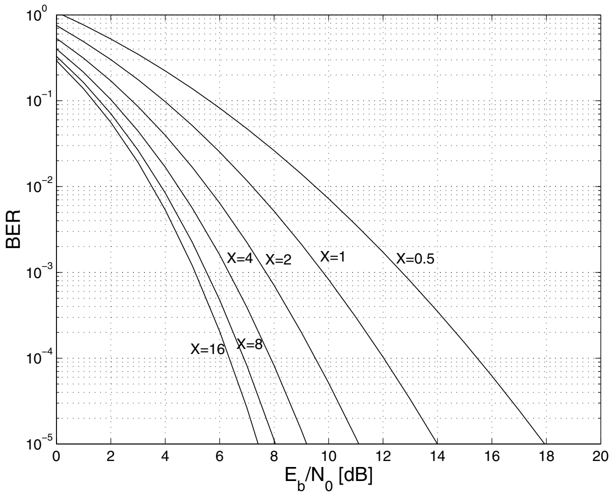

Fig. 6. Bit error rates (Union Bounds) for correlated Rayleigh fading ($X = 0.5, 1, 2, 4, 8, 16$) and the WH code with $M = 64$ and intra code word interleaving.

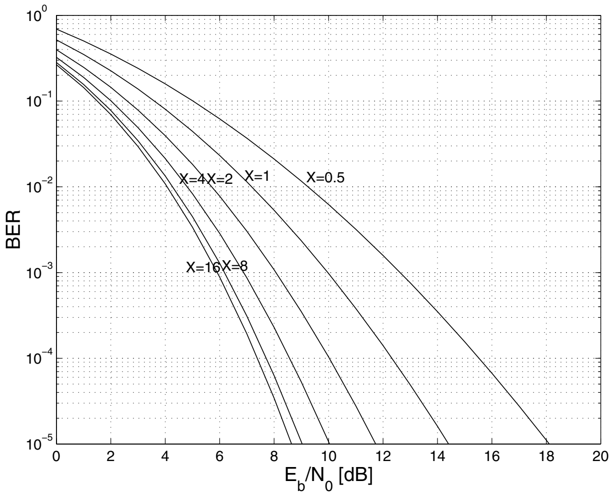

Fig. 7. Bit error rates (Union Bounds) for correlated Rayleigh fading ($X = 0.5, 1, 2, 4, 8, 16$) and the WH code with $M = 32$ and intra code word interleaving.

different channels, the system design must take care of these degradations.

6. Application to related problem

We have chosen two simple examples to apply our method to calculate the error probability for a correlated Rayleigh fading channel. The repetition code – which is just the standard form of diversity – is very simple because it only consists of two code words. Therefore, it has only one error event and the bit error probability equals the pair error probability. The Walsh-Hadamard code of length M has M code words and $M - 1$ error events for a fixed transmitted code word. This code is easy to analyse because it has constant weight $M/2$ and therefore any two code words differ exactly in $d_H = M/2$ positions. Thus we have to carry out our analysis only for a fixed $L = M/2$. We have to point out that the pairwise error probability does not depend only on L but also of the positions $i_1, ..., i_L$ where the two code words differ. We have made use of the union bound technique for this code. This can only be applied for a finite and simple weight spectrum of the code.

For convolutional codes, we would run into troubles if we would apply union bounds for the correlated fading channel. The sum of the union bound is infinite and we have to consider error events with increasing Hamming distance and increasing error coefficients. For correlated fading, the diversity of the channel is limited. The pair error probabilities would not get smaller for increasing distance and the union bound would diverge. However, our analysis can give us some practical hints for the choice of the code. Let us compare e.g. a memory 6 convolutional code with rate $R_c = 1/2$ and free distance $d_{free} = 10$ with another memory 6 convolutional code with rate $R_c = 1/4$ and free distance $d_{free} = 20$ (these are the parameters of the best known codes of memory 6). Assume an equivalent independent channel like shown in Fig. 1. We can conclude from the figure that the stronger code can take advantage from the higher diversity degree only if the selectivity X of the channel is very high. For moderate values like $X = 4$ the useful diversity is the same and there is no coding gain because the product $d_{free}R_c$ is also the same. For a more careful analysis, one should the compare the weight factors. Weight factor considerations could then also provide arguments for considering lower memories.

The equivalent independent channel may also give valuable hints for the choice of the appropriate interleaver size for a given code. This method also provides us with an analytical tool instead of simulations to study effects of imperfect interleaving like it has been done for the Digital Video Broadcasting (DVB) system [9] in [10]. It it obvious that our analysis also applies for correlations in time direction. The frequency autocorrelation function must then be replaced by the time autocorrelation function given by the Doppler spectrum. For mixed time and frequency interleaving (like applied in the Digital Audio Broadcasting system DAB [11]), the generalization is straightforward: The channel samples in time and frequency direction have to be written in a single vector and the autocorrelation matrix for this stochastic vector must be calculated.

7. Conclusions

We have presented a method for the analytical evaluation of the pairwise error probability for coded transmission in a correlated Rayleigh fading channel. It is given in a closed form as an integral over a finite interval which can be evaluated numerically. The relevant quantities are the eigenvalues λ_i of the frequency autocorrelation matrix. They can be regarded as the branch power distribution of an *equivalent independently fading diversity channel*. Even though applied only for the special case of a MC-CDMA system, the method can be easily applied to similar problems where correlated fading occurs, e.g. coded transmission with insufficient time and/or frequency interleaving, frequency hopping, and other related topics.

Appendix

A1. Proof of the alternative form of the gaussian probability integral

In this Appendix, we present a simple geometrical proof of the alternative form of the Gaussian probability integral used above. We view the one-dimensional problem of pairwise error probability as two-dimensional and introduce polar coordinates. We consider the noise as a Gaussian random variable with mean zero and variance $\sigma^2 = 1$ and ask for the probability that the random variable exceeds a positive real value that we call x. The well-known answer is given by the Gaussian probability integral

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\xi^2\right) d\xi. \qquad \text{(A1)}$$

This probability does not change if we consider also the noise of same variance in the 2nd dimension. The error threshold is now a straight line parallel to the axis of the 2nd dimension and the probability is given by

$$Q(x) = \int_x^\infty \left(\int_{-\infty}^\infty \frac{1}{2\pi} \exp\left(-\frac{1}{2}(\xi^2+\eta^2)\right) d\eta \right) d\xi. \qquad \text{(A2)}$$

This integral can be written in polar coordinates (r, ϕ) as

$$Q(x) = \int_{-\pi/2}^{\pi/2} \left(\int_{x/\cos\phi}^\infty \frac{r}{2\pi} \exp\left(-\frac{1}{2}r^2\right) dr \right) d\phi. \qquad \text{(A3)}$$

The integral over r can immediately be solved to give

$$Q(x) = \int_{-\pi/2}^{\pi/2} \frac{1}{2\pi} \exp\left(-\frac{1}{2}\frac{x^2}{\cos^2\phi}\right) d\phi. \qquad \text{(A4)}$$

A simple symmetry argument now leads to the desired form of $\frac{1}{2}\mathrm{erfc}(x) = Q(\sqrt{2}x)$.

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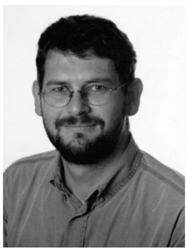

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