

Diversity Spectra of Dense Antenna Arrays

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Overview and Questions

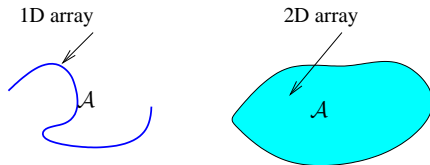
- How much diversity can be achieved by a given antenna array geometry (aperture) ?
- Consider *dense antenna arrays* (apertures) with spatial correlations
- Calculation of the **eigenvalue spectrum** of the autocorrelation kernel
- This **diversity spectrum** depends on the aperture and the power azimuth spectrum (PAS)

Goal: Quantitative statements about diversity gains of a given combination of **aperture geometry** and **PAS**

The Concept of a Dense Array (Aperture) \mathcal{A}

Visual Concept: Continuous limit of an array densely covered by infinitely many small sensors

Mathematical Concept: Continuous stochastic process (e.m. field) on a geometrical object \mathcal{A} (1D or 2D) in the plane

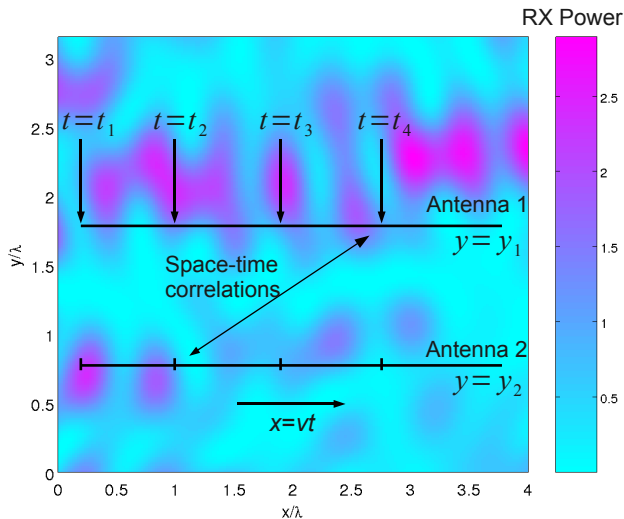


↪ Each **aperture** has its specific diversity properties

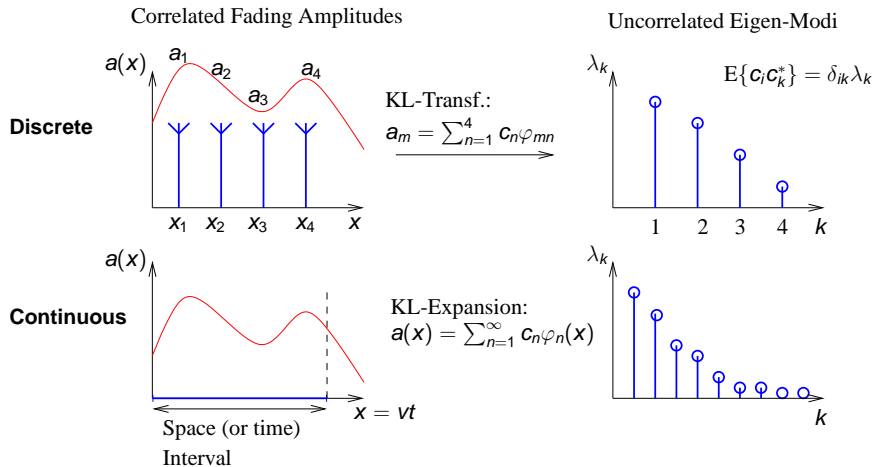
Goal: Calculate the diversity of a given array

Time Variance of Fading = Scaled Space Variance

Aperture \mathcal{A} for 2 antennas on a moving vehicle: 2 parallel line segments (2 ULAs)

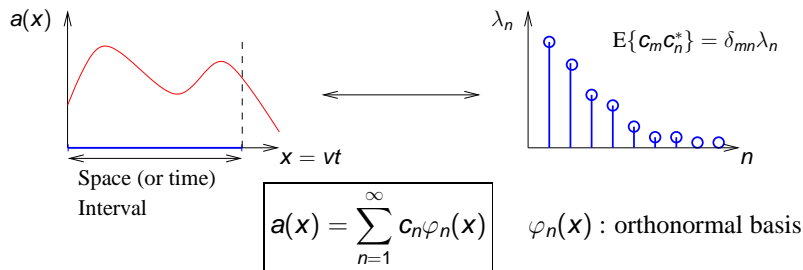


Karhunen-Loève (KL) Expansion



$\{\lambda_n\}$: Eigenvalue spectrum of the autocorrelation matrix (discrete case) or integral kernel (continuous case) of the fading

Karhunen-Loève (KL) Expansion - Interpretation

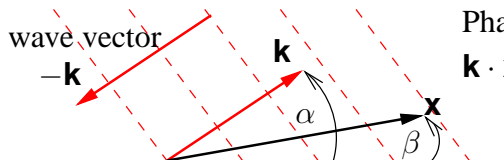


Interpretation:

- Equivalent uncorrelated diversity branches, each corresponding to one term in the sum
- $\lambda_n = E \left\{ |c_n|^2 \right\} \leftrightarrow$ power gain of branch number n
- We shall see: For a finite array \mathcal{A} , only a finite number of terms in the sum are relevant (\leftrightarrow eigenvalues λ_n of significant size)
- \rightsquigarrow Finite degree of diversity for a finite array \mathcal{A}

Spatial 2D Fading Process

Wave (wavelength=1) impinging from direction α :



Phase at point \mathbf{x} :

$$\mathbf{k} \cdot \mathbf{x} = 2\pi r \cos(\alpha - \beta)$$

$$\mathbf{k} = 2\pi \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \quad \mathbf{x} = r \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix}$$

Spatial fading process = superposition of many plane waves :

$$a(\mathbf{x}) = \int_{-\pi}^{\pi} \hat{a}(\alpha) e^{j2\pi r \cos(\alpha - \beta)} d\alpha$$

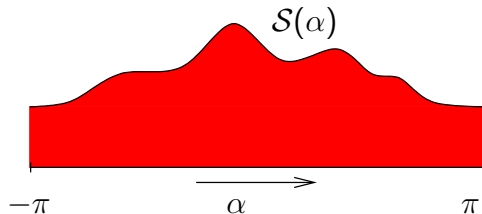
$\hat{a}(\alpha)$: scattering amplitude of wave from direction α

Spatial Fading Statistics

Assumption: *Uncorrelated Scattering* (US):

$$E \{ \hat{\mathbf{a}}(\alpha) \hat{\mathbf{a}}^*(\alpha') \} = \mathcal{S}(\alpha) \delta(\alpha - \alpha')$$

$\mathcal{S}(\alpha)$ = Power Azimuth Spektrum (PAS): **Known from the model**



US \Leftrightarrow Translational symmetry of the autocorrelation:

$$E \{ \mathbf{a}(\mathbf{x}) \mathbf{a}^*(\mathbf{x}') \} = \rho(\mathbf{x} - \mathbf{x}')$$

$$\rho(\mathbf{x}) = \int_{-\pi}^{\pi} e^{j2\pi r \cos(\alpha - \beta)} \mathcal{S}(\alpha) d\alpha$$

Fourier Expansions and the PAS Matrix $\tilde{\mathbf{R}}$

$$\hat{\mathbf{a}}(\alpha) = \sum_{n=-\infty}^{\infty} \hat{a}_n e^{j\alpha n} \quad \text{and} \quad \tilde{\mathbf{a}}_n = 2\pi \hat{\mathbf{a}} = \int_{-\pi}^{\pi} e^{-j\alpha n} \hat{\mathbf{a}}(\alpha) d\alpha$$
$$\mathcal{S}(\alpha) = \sum_{n=-\infty}^{\infty} \hat{s}_n e^{j\alpha n} \quad \text{and} \quad \tilde{\mathbf{s}}_n = 2\pi \hat{\mathbf{s}} = \int_{-\pi}^{\pi} e^{-j\alpha n} \mathcal{S}(\alpha) d\alpha$$

Uncorrelated scattering \implies The *PAS (correlation) matrix* $\tilde{\mathbf{R}}$ with elements

$$\tilde{R}_{mn} \triangleq \text{E} \{ \tilde{\mathbf{a}}_m \tilde{\mathbf{a}}_n^* \} = \tilde{\mathbf{s}}_{m-n}$$

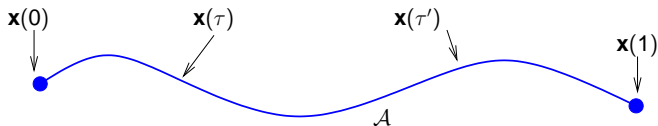
is Toeplitz.

Equivalent descriptions of the scattering environment:

$$\mathcal{S}(\alpha) \longleftrightarrow \tilde{\mathbf{R}}$$

The KLE on a 1D Array (Curve in the Plane)

$\mathbf{x}(\tau)$: Parametrisation of the curve \mathcal{A} . Parameter $\tau \in [0, 1]$



$\mathbf{a}(\tau) \triangleq \mathbf{a}(\mathbf{x}(\tau))$: Fading process on the curve

KL Expansion of the fading process on the curve:

$$\mathbf{a}(\tau) = \sum_{n=1}^{\infty} \mathbf{c}_n \varphi_n(\tau)$$

$$\mathbf{c}_n = \int_0^1 \varphi_n^*(\tau) \mathbf{a}(\tau) d\tau, \quad \mathbb{E} \{ \mathbf{c}_m \mathbf{c}_n^* \} = \lambda_n \delta_{mn} \text{ with}$$

eigenvalues λ_n and eigenfunctions $\varphi_n(\tau)$ of the integral kernel

$$R(\tau, \tau') = \mathbb{E} \{ \mathbf{a}(\tau) \mathbf{a}^*(\tau') \} = \rho(\mathbf{x}(\tau) - \mathbf{x}(\tau'))$$

How to Solve the Eigenvalue Problem: Transformation

Non-orthogonal expansion of the spatial fading process:

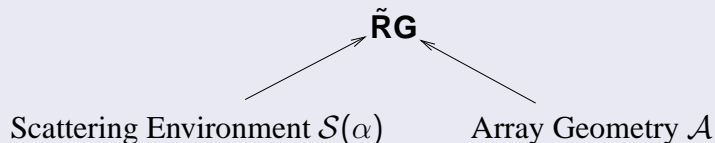
$$\mathbf{a}(\mathbf{x}) = \sum_{n=-\infty}^{\infty} \tilde{a}_n \mathbf{v}_n(\mathbf{x}) \quad , \quad \mathbf{v}_n(\mathbf{x}) = e^{j\beta n} j^n J_n(2\pi r)$$

Define Gram matrix \mathbf{G} with elements

$$\mathbf{G}_{mn} = \int_0^1 \mathbf{v}_m^*(\tau) \mathbf{v}_n(\tau) d\tau, \quad \left(\mathbf{v}_n(\tau) \triangleq \mathbf{v}_n(\mathbf{x}(\tau)) \right)$$

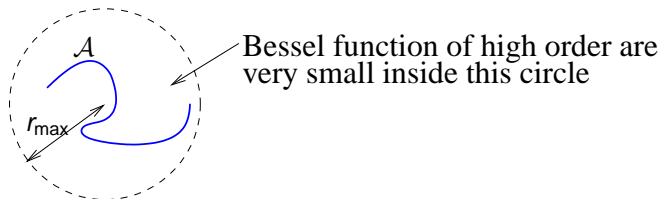
Theorem

The kernel $R(\tau, \tau')$ has the same eigenvalues as the infinite-dimensional matrix



How to Solve the Eigenvalue Problem: Truncation

Assume that \mathcal{A} is inside a finite circle



$$\Rightarrow |\mathbf{G}_{mn}| \leq \int_0^1 |\mathbf{J}_m(2\pi |\mathbf{x}(\tau)|)| |\mathbf{J}_n(2\pi |\mathbf{x}(\tau)|)| d\tau \rightarrow 0$$

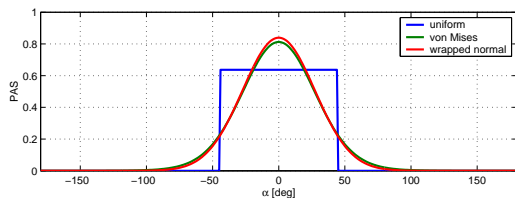
(exponentially fast) for $|m|, |n| \rightarrow \infty$

Theorem

When the infinite matrix $\tilde{\mathbf{R}}\mathbf{G}$ is replaced by a truncated finite one, the effect on the eigenvalues is exponentially small.

\rightsquigarrow Solve the EVP numerically for the truncated matrix

PAS Models $\mathcal{S}(\alpha)$ (and $\mathcal{S}(\alpha - \alpha_0)$ for AoA α_0)



1 Uniform with width Δ

$$\mathcal{S}(\alpha) = \text{rect}\left(\frac{\alpha}{\Delta}\right) \Rightarrow \tilde{\mathcal{S}}_n = \text{sinc}\left(n\frac{\Delta}{2\pi}\right)$$

2 von-Mises with dispersion κ^{-1}

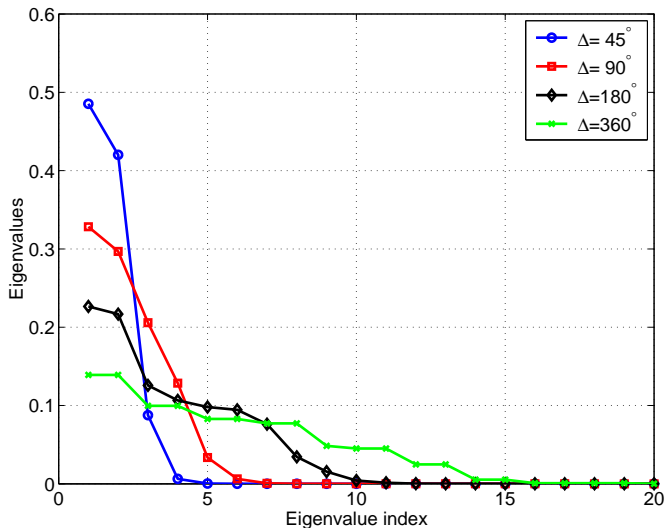
$$\mathcal{S}(\alpha) = \frac{\exp(\kappa \cos \alpha)}{2\pi I_0(\kappa)} \Rightarrow \tilde{\mathcal{S}}_n = \frac{I_n(\kappa)}{I_0(\kappa)}$$

3 Wrapped Normal with dispersion σ^2

$$\mathcal{S}(\alpha) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{k=-\infty}^{\infty} e^{-\frac{(\alpha-2\pi k)^2}{2\sigma^2}} \Rightarrow \tilde{\mathcal{S}}_n = e^{-\frac{1}{2}\sigma^2 n^2}$$

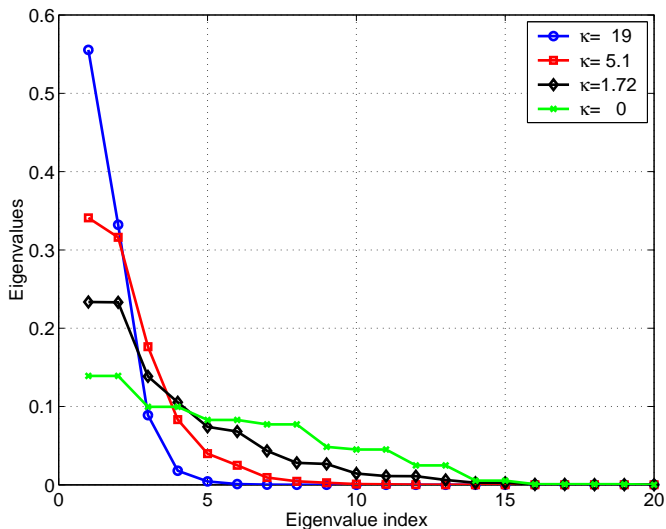
Calculated Diversity Spectra

Uniform circular array (UCA) of radius $r = 1$, uniform PAS



Calculated Diversity Spectra

Uniform circular array (UCA) of radius $r = 1$, von Mises PAS



Diversity Measure

Ivrlač, Nосsek 2003; Muharemovic et al. 2004/2008

Attempt to summarise the main properties of $\{\lambda_n\}$ by a single number ω

For L diversity branches:

$$\omega \triangleq \frac{\left(\sum_{i=1}^L \lambda_i\right)^2}{\sum_{i=1}^L \lambda_i^2}$$

$\omega = L$ for uncorrelated branches and $\omega = 1$ for 100% correlation

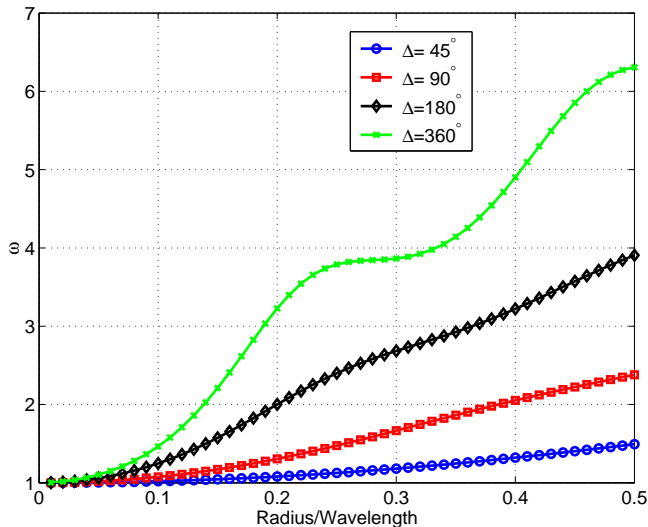
Lozano et al. 2003; Muharemovic 2004/2008: ω determines the capacity slope of a MIMO system in the low power regime

We extend the definition to a continuous aperture:

$$\omega \triangleq \frac{\left(\sum_{i=1}^{\infty} \lambda_i\right)^2}{\sum_{i=1}^{\infty} \lambda_i^2}$$

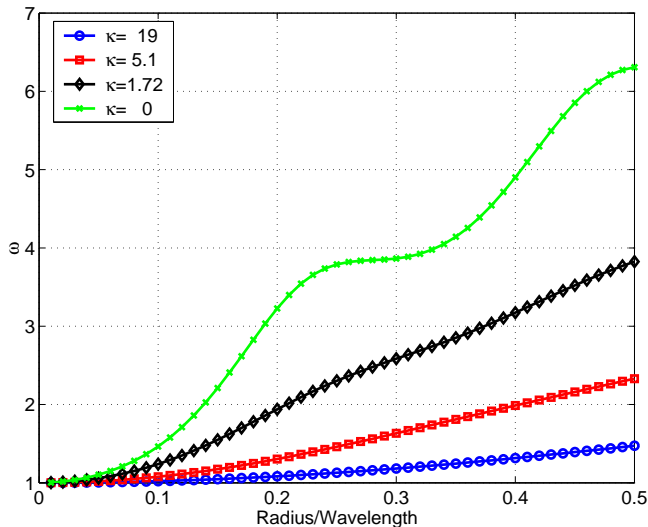
Diversity Measure as a Function of the Array Size

Uniform circular array (UCA), uniform PAS



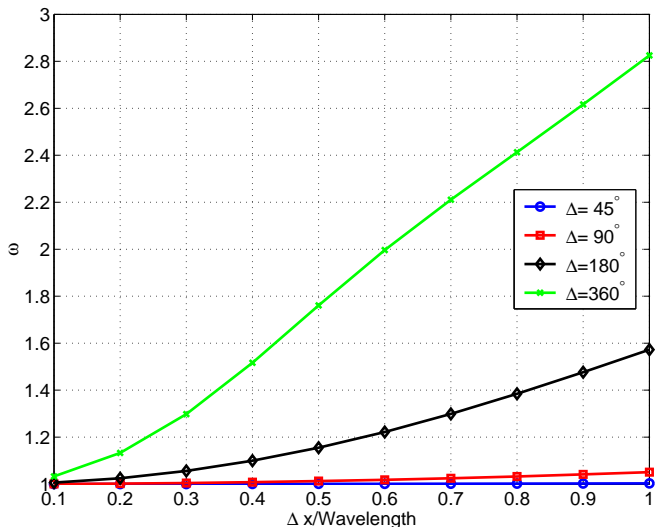
Diversity Measure as a Function of the Array Size

Uniform circular array (UCA), von Mises PAS



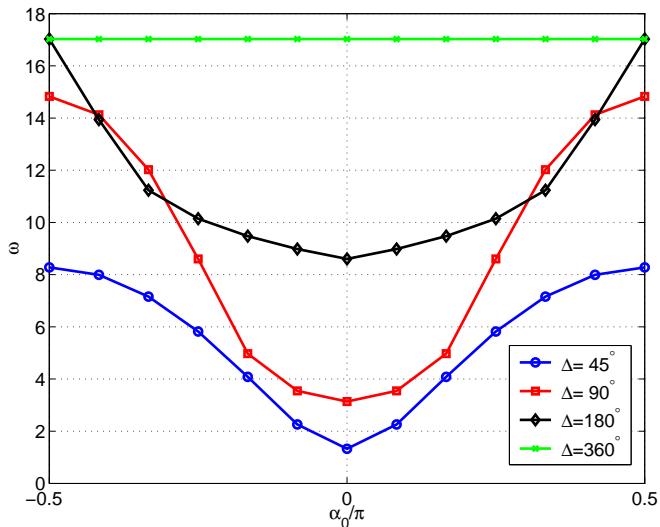
Diversity Measure as a Function of the Array Size

Uniform linear array (ULA), uniform PAS, AoA $\alpha_0 = 0$ (signal || array)



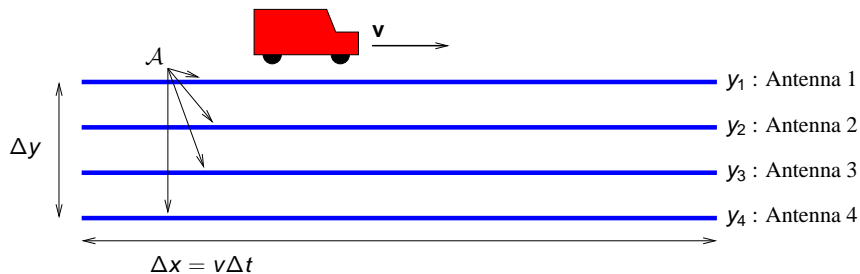
Diversity Measure as a Function of the AoA α_0

Large uniform linear array (ULA) of size $\Delta x = 10$, uniform PAS



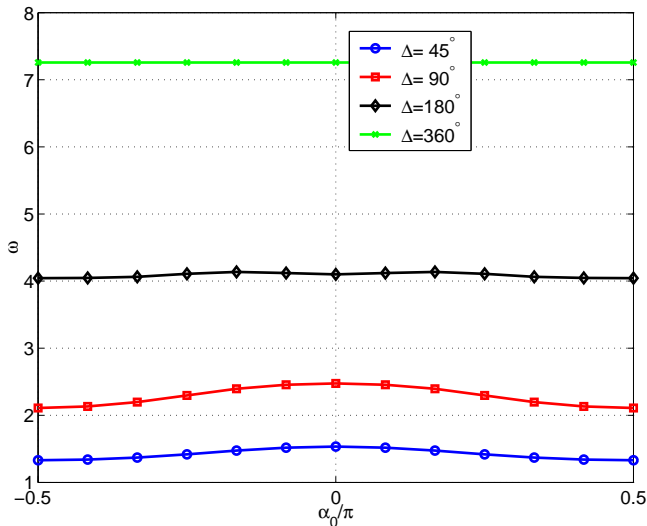
4 Parallel Dense ULAs

⇔ 4 Antennas on a Moving Vehicle



Diversity Measure as a Function of the AoA α_0

4 parallel ULAs of aperture size $\Delta x = 1$, $\Delta y = 1$, uniform PAS



- Time interleaving and spatial diversity are entangled \Rightarrow antenna separation and time interleaving depth should be jointly designed
- ULAs are very sensitive against small PAS opening angles. This may more frequently occur at the base station, less frequently at the mobile
- 2D structures are favourable \rightsquigarrow The **antenna array** should be mounted *traverse* to the direction of motion and *not parallel*

- We have presented a method to calculate diversity spectra for general dense arrays
- Dense (sensor) arrays are the continuous limit of discrete (antenna) arrays
- Diversity spectra allow to calculate bit error curves (see H.S., InoWo 2010)
- Diversity measures determine the MIMO capacity in the low power regime