Adjacent Channel Interference Effects in OFDM Systems with Imperfect Anti-Aliasing Filtering

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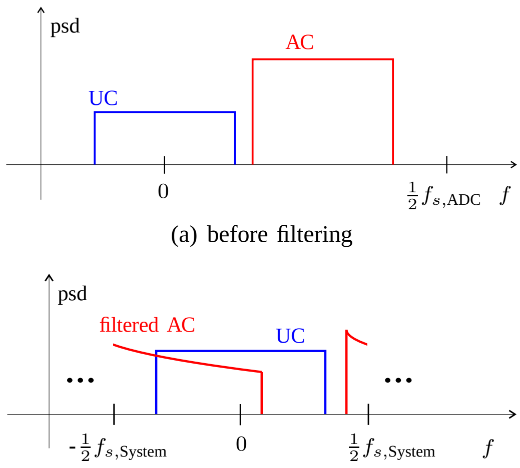
***Abstract*—We examine the BER of an OFDM signal that is interfered by an imperfectly filtered adjacent OFDM channel as aliasing component. For this purpose, we interpret the overlaying signal as additive noise that is shaped by the power response of the anti-aliasing filter. We propose a discrete channel model and derive a formula for the error event probabilities. With this expression, the BER can be solved numerically for arbitrarily distributed colored Gaussian noise. For a special class of power spectral densities, we show the analytical solution with the dynamic range of the noise process as parameter. We examine the influence of the dynamic range on the BER and show on an example that our proposed method provides a good prediction for anti-aliasing filtering with LMS designed FIR filters.**

I. Introduction

The specification of receiver requirements is a crucial part in the development process of broadcast receivers. An accurate prediction of the performance of receivers is needed. Amongst others, the necessary suppression of adjacent channels is an important aspect.

In OFDM broadcast systems, independent channels are radiated from various base stations. Due to their different distance to the receiver and varying propagation conditions, their receipt signal power strength can differ considerably. Figure 1(a) illustrates an OFDM user channel in the complex baseband whose receiving power is dominated by its adjacent channel.

Channel filtering is needed at the receiver. Analog channel filters, however, cannot suppress the adjacent channel sufficiently. Therefore the signal is oversampled by the analog-to-digital converter (ADC) and the task of filtering is shifted to the digital domain. The sampling rate is decimated from the sampling rate of the ADC $f_{s,\text{ADC}}$ to a specific sampling rate of the system $f_{s,\text{System}}$. Since the adjacent channel cannot be filtered out entirely, aliasing occurs from this sampling rate conversion, that degrades the quality of reception. Figure 1(b) shows a possible constellation of the user channel and the imperfectly filtered adjacent channel as aliasing component after downsampling.



(a) before filtering

(b) after filtering and down sampling

Fig. 1: Illustration of the spectra of the user channel (UC) and the adjacent channel (AC) in the complex baseband.

In this paper, we investigate the bit error probabilities due to such broadband interference. In Section II, we analyze the disturbing influence of an OFDM signal that overlays another OFDM signal. In Section III, we propose a discrete channel model to describe filtered or ”colored” OFDM interference. With this model, we derive a formula for the error event probabilities and give an analytically solution for a certain class of power spectral densities (psd) in Section IV. In Section V, we discuss our findings and give numerical results to confirm our proposed method. Finally we conclude in Section VI.

II. Interfering OFDM as Additive Noise

We investigate the influence of an OFDM channel that interferes another OFDM channel completely without previous filtering. All subcarriers of an OFDM signal are radiated with the same power. Therefore its power spectral density is constant or "white" over its whole bandwidth. It is also a common assumption that the distribution of the amplitudes of an OFDM signal is Gaussian [1], where this thought is supported by the central limit theorem.

However, it cannot be concluded that an OFDM channel, that interferes another OFDM channel, can be necessarily interpreted as AWGN. If, for example, the subcarrier grid of the user channel and the aliasing channel match exactly and both signals are perfectly synchronized in the time domain, then a single modulation symbol of the user channel is distorted additively by another single modulation symbol of the adjacent channel. The influence of this – obviously non-Gaussian – distortion, however, is negligible for reasonable signal-to-interferer ratios.

Due to the specification of the OFDM systems, we can expect the channels to be frequency synchronized. Time synchronization, however, is not very likely since both signals are radiated from different base stations and come into the receiver over independent paths.

As a worst case estimate, we investigate interference with time offset $\tau = \frac{1}{2}T_s$ between both signals, where T_s is the useful OFDM symbol duration. Figure 2 shows the simulated BER versus E_b/I_0 for this case. The BER versus E_b/N_0 for AWGN channels are plotted as well. It can be seen that the BER from aliasing OFDM can be reasonably upper bounded by the BER from AWGN.

In the following, we will use AWGN as *worst case approximation* for interfering OFDM but we will still keep in mind that its influence is not exactly equivalent and that the error probabilities act as upper bounds.

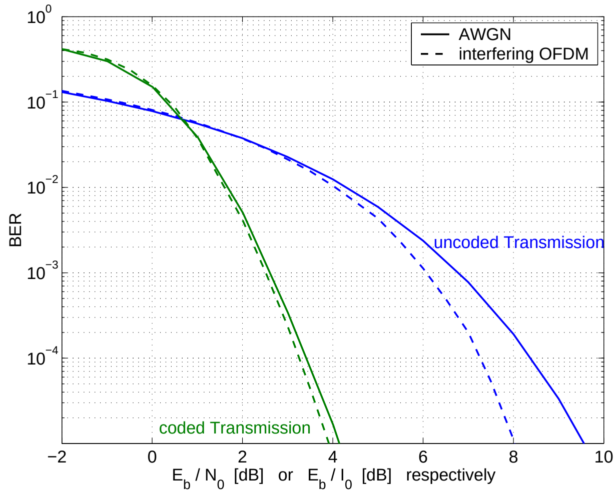


Fig. 2: Comparison of the BER for AWGN and superimposing OFDM with time offset $\tau = \frac{1}{2}T_s$. (BPSK, Code rate $r_c = \frac{1}{2}$)

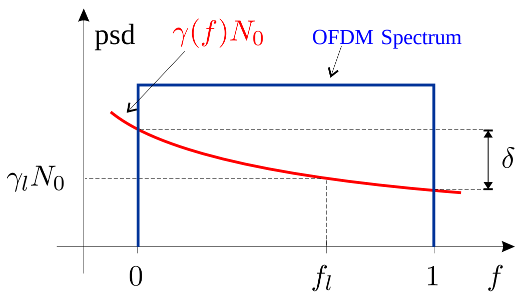


Fig. 3: Illustration of an OFDM user spectrum and the psd of the colored noise process over the normalized frequency. (δ is the *dynamic range*, defined in Equation (11))

III. Channel Model

In the next paragraph, we propose a discrete channel model for data transmission with additive colored Gaussian noise, that can be used to model the influence of an interfering OFDM channel that is filtered before downsampling.

At the transmitter, data bits are mapped to complex symbols and, following the OFDM principle, N_c symbols are radiated simultaneously over orthogonal subcarriers within the bandwidth $B = N_c T_s^{-1}$.

We assume channel filtering with non-constant attenuation in the stopband. In that case, the psd of the adjacent channel is no longer white but "colored". We name its *mean* power spectral density N_0.

In our channel model, the OFDM interferer is replaced by AWGN. To model the shape of its psd, we define a frequency dependent power weighting factor $\gamma \in \mathbb{R}^+$. So the noise psd at frequency f_l is $\gamma_l N_0$ with $\gamma_l = \gamma(f_l)$. For convenience, we normalize the frequency so that $f = 0$ represents the left edge and $f = 1$ represents the right edge of the OFDM bandwidth. Figure 3 shows an example of frequency dependent noise in an OFDM user spectrum.

As an approximation, we postulate that the psd shall change only slowly with the frequency. Then we imply that over the bandwidth of one single subcarrier, the noise process can still be considered as white. A symbol that is transmitted at frequency f_l, is then distorted by AWGN with noise power spectral density $\gamma_l N_0$.

The frequency interleaver maps every complex symbol x_i to a certain frequency f_i. So if we assume ideal interleaving, this frequency can be regarded as random variable, that is uniformly distributed over the interval $[0, 1]$.

This leads to our proposed discrete channel model in Figure 4. The sequence $\mathbf{x}$ of K complex data symbols is distorted by additive noise $\mathbf{n} = (n_1, \ldots, n_K)^T$. This noise vector $\mathbf{n}$ results from the multiplication of white Gaussian noise $\mathbf{w} = (w_1, \ldots, w_K)^T$ with variance $E\{w_i^2\} = \sigma^2 = N_0/2$ and the matrix of weighting factors $\mathbf{L} = \mathrm{diag}(\sqrt{\gamma_1}, \ldots, \sqrt{\gamma_K})$. The factors $\sqrt{\gamma_i}$ can be found via the transformation of the uniformly distributed random variable f through $\gamma_i = \gamma(f_i)$. The input to the receiver is

$$\mathbf{y} = \mathbf{x} + \mathbf{L}\mathbf{w} \ . \tag{1}$$

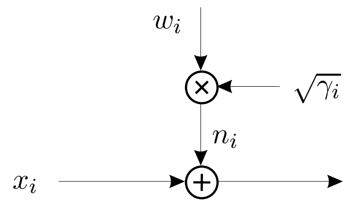


Fig. 4: Proposed channel model for additive colored Gaussian noise

IV. Bit Error Analysis

In this section, we derive analytical bounds for the bit error ratio for transmission of BPSK symbols over the channel model from Section III.

For convolutional codes the BER can be upper bounded by the union bound [2]

$$BER \leq \sum_{d=d_{free}}^{\infty} c_d P_d \ , \tag{2}$$

where P_d is the probability that a wrong sequence of distance d is chosen by the Maximum Likelihood Sequence Estimator (MLSE). The so-called error coefficients c_d are weighting factors, that are tabulated for many codes. In the next paragraph, the error event probabilities P_d will be derived.

Instead of the channel model of Equation (1) we use the equivalent AWGN model

$$\mathbf{L}^{-1}\mathbf{y} = \mathbf{L}^{-1}\mathbf{x} + \mathbf{w} \tag{3}$$

for further analysis The MLSE finds the sequence that maximizes the scalar product

$$\left\langle \mathbf{L}^{-1}\mathbf{x}, \mathbf{L}^{-1}\mathbf{y} \right\rangle = \sum_{i=1}^{K} \frac{1}{\gamma_i} x_i y_i \ . \tag{4}$$

Equation (4) has similarities to the sequence estimation in fading channels. Here the MLSE maximizes $\langle \mathbf{Cx}, \mathbf{Cy} \rangle$, where $\mathbf{C}$ is the matrix of channel coefficients. We follow the method described in [3] and find the pairwise error probabilities that the receiver decides erroneously for $\hat{\mathbf{x}}$ instead of $\mathbf{x}$

$$P(\mathbf{x} \to \hat{\mathbf{x}} | \mathbf{L}) = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{1}{4N_0} \| \mathbf{L}^{-1}\mathbf{x} - \mathbf{L}^{-1}\hat{\mathbf{x}} \|^2} \right) \ , \tag{5}$$

under the condition that $\mathbf{L}$ has occurred. While introducing the polar representation of the complementary error integral [4] and considering that all γ_i are independent and belong to the same distribution, it follows that

$$P_d = \frac{1}{\pi} \int_0^{\pi/2} E_\gamma \left\{ \exp\Big(- \frac{E_s}{\gamma N_0 \sin^2 \phi} \Big) \right\}^d \mathrm{d}\phi \ , \tag{6}$$

with E_γ the expectation value over γ and E_s the symbol energy. This expectation value and the finite integral can be solved numerically for any shape of the psd or if e.g. the adjacent channel overlaps the user channel only partly.

In the next paragraph, we will give an example for a class of functions, where $E_\gamma\{\cdot\}$ in Equation (6), namely

$$E_\gamma\{\cdot\} = \int_{\gamma_{min}}^{\gamma_{max}} p(\gamma) \exp\Big(- \frac{E_s}{\gamma N_0 \sin^2 \phi} \Big) \mathrm{d}\gamma \ , \tag{7}$$

can be found analytically. As psd of the colored noise process we choose the function

$$\gamma(f) = \frac{a}{f + b} \tag{8}$$

with $a, b \in \mathbb{R}^+$ free but constant parameters.

The uniformly distributed random variable f is transformed to the random variable γ by Equation (8). The probability density function (pdf) $p(\gamma)$ of this transformed random variable is defined by $p(\gamma)\mathrm{d}\gamma = -\mathrm{d}f$, where the minus sign is used because $\gamma(f)$ is monotonically decreasing [5]. So the pdf can be found as

$$p(\gamma) = \frac{a}{\gamma^2} \ . \tag{9}$$

With this pdf the expectation value from Equation (7) and the error event probabilities from Equation (6) can be solved

$$P_d = \frac{1}{\pi} \int_0^{\pi/2} \Bigg[\Bigg(\frac{a N_0 \sin^2 \phi}{E_s} \exp\Big(- \frac{E_s}{a N_0 \sin^2 \phi} b \Big) - \exp\Big(- \frac{E_s}{a N_0 \sin^2 \phi} (b+1) \Big) \Bigg) \Bigg]^d \mathrm{d}\phi \ . \tag{10}$$

We define the dynamic range δ of the colored noise process of function (8), as the power ratio between the highest and lowest value of γ (see also Figure 3)

$$\delta = \frac{\gamma_{max}}{\gamma_{min}} \ . \tag{11}$$

To keep the mean psd N_0 of the colored process independent of a and b, we normalize

$$\int_0^1 \gamma(f) \mathrm{d}f \stackrel{!}{=} 1 \ . \tag{12}$$

With Equations (11) and (12) the error event probabilities

$$P_d = \frac{1}{\pi} \int_0^{\pi/2} \left[\frac{1}{\frac{E_s}{N_0 \sin^2 \phi} \ln \delta} \cdot \left(\exp\left(-\frac{E_s}{N_0 \sin^2 \phi} \frac{\ln \delta}{\delta - 1} \right) - \exp\left(-\frac{E_s}{N_0 \sin^2 \phi} \frac{\delta \ln \delta}{\delta - 1} \right) \right) \right]^d \mathrm{d}\phi \tag{13}$$

are completely described by the dynamic range δ and the average signal-to-noise ratio E_s/N_0. These are both values that can be measured in a real system.

V. Results

In this section, we analyze the BER from Equation (2) with the error event probabilities from Equation (13). For uncoded transmission, the first error event probability P_1 gives a good approximation. For coded transmission with code rate $r_c = \frac{1}{2}$ and the generator polynomial $g = (133, 171)_{\mathrm{oct}}$, the error coefficients c_d can be found in [6].

A. Effect of the dynamic range δ

The BER is dependent on the dynamic range δ of the colored noise process. Figure 5 shows the analytical BER for different dynamic ranges versus E_b/N_0, that were also verified by Matlab simulations. As expected for $\delta = 0.01$ dB, the BER is equal to that of an AWGN channel (see Figure 2), because the psd is flat over the frequency.

It is interesting to note that with increasing dynamic range, the BER declines for high E_s/N_0, whereas it improves for low E_s/N_0. This can be explained with Figure 6, where an OFDM spectrum and colored noise with $\delta = 2$ dB and $\delta = 10$ dB are shown. In 6(a), the SNR is $E_s/N_0 = 7$ dB. For a noise process with $\delta = 2$ dB, all subcarriers have a high SNR and so most of them are received correctly. For $\delta = 10$ dB on the other hand, there are some subcarriers on the left side of the OFDM spectra that are corrupted by very intense noise and therefore are most likely wrong. In 6(b), the SNR

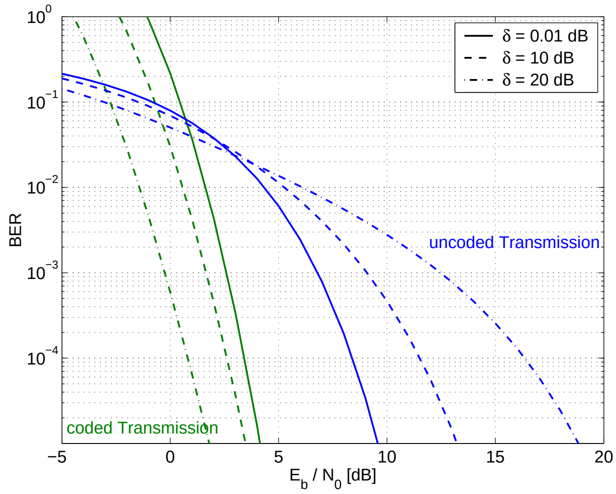
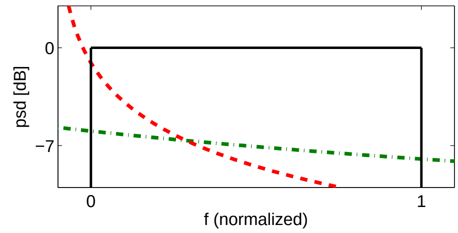
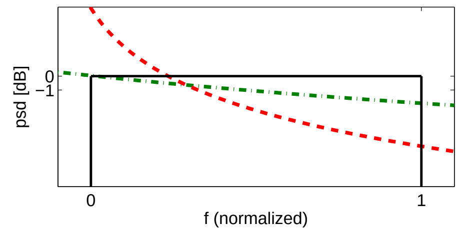


Fig. 5: Calculated BER for Equation (2) with error event probabilities from Equation (13).



(a) $E_s/N_0 = 7$ dB



(b) $E_s/N_0 = 1$ dB

Fig. 6: Illustration of colored noise with dynamic range $\delta = 2$ dB (dash-dot) and $\delta = 10$ dB (dash) in an OFDM spectrum with high SNR (a) or low SNR (b).

is $E_s/N_0 = 1$ dB. All subcarriers that are distorted by colored noise with $\delta = 2$ dB have a low SNR and so many of them are received incorrectly. For a noise process with $\delta = 10$ dB however, the carriers on the right end of the OFDM spectra are corrupted by very weak noise and therefore have a low error probability.

So if we compare two interfering OFDM channels with identical mean psd but different dynamic ranges, it can not directly be determined if the channel with higher or lower dynamic range is more destructive. This depends also on the actual mean signal-to-interferer ratio after filtering.

B. Anti-Aliasing Filtering

Now we test our proposed method on a realistic filter design. For this, we simulated an adjacent OFDM channel that was insufficiently filtered out and added to the user channel. We used a linear phase FIR filter with filter order $N = 50$, that was designed to meet the least mean-square (LMS) error approximation in the frequency domain. The power response of the digital filter is depicted in Figure 7, where the spectra of the OFDM user and adjacent channel are indicated as well. The attenuation of the filter is 25 dB on the left edge and 49 dB on the right edge of the adjacent channel. For our analysis, we substitute the stopband of the filter inside the user channel with our proposed function (8) with dynamic range $\delta = 24$ dB. It can be seen in Figure 7 that it fits to the power response as an approximation.

Figure 8 shows the BER versus E_b/N_0 for the simulation with the filter design. The calculated BER from Equations (2) and (13) with $\delta = 24$ dB and the results of an AWGN channel are given as well. One can see that the BER for the simulation with the filtered channel and the BER for AWGN differ considerably. Our model, however, gives a good approximation. With this example we showed that our model can be used to estimate the BER of OFDM signals that are interfered by filtered adjacent channels after downsampling.

VI. Conclusion

We investigated the effect on an OFDM user signal that is interfered by an insufficiently anti-aliasing filtered adjacent OFDM channel. We showed that frequency-flat OFDM interference is not equal to additive white Gaussian noise, but its BER can be reasonably upper bounded by AWGN as a worst case approximation. A discrete channel model was proposed for all additive Gaussian noise processes with arbitrary psd. A general formula was derived for the error event probabilities, that can be solved numerically, if the psd of the noise process is not known. For a special class of power spectral densities, the analytical solution was given, that fits to anti-aliasing filtering with digital FIR filters, that meet the LMS approximation in the frequency domain. We would like to point out that this expression depends only on the mean energy and the dynamic range of the noise process and can therefor also be used for other noise processes such as flicker noise. Finally, we verified our proposed method by simulations.

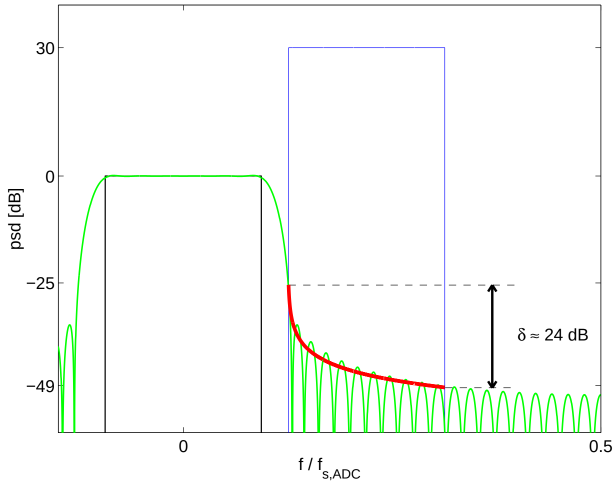


Fig. 7: Power response of the digital anti-aliasing FIR filter. The stopband is approximated by the proposed function (8) with dynamic range $\delta = 24$ dB.

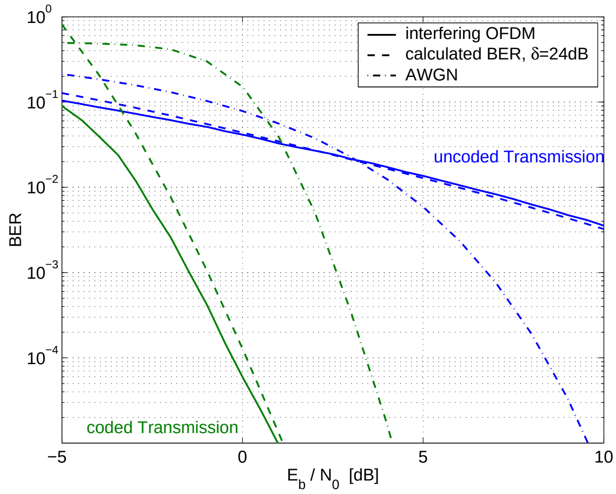


Fig. 8: BER for a signal that is interfered by a filtered OFDM signal and the calculated BER with $\delta = 24$ dB. BER for AWGN is given as reference.

References

[1] C. Gandy. *DAB: an introduction to the Eureka DAB System and a guide to how it works*, R&D White Paper WHP 061, Research and Development, British Broadcasting Corporation (BBC), June 2003

[2] Andrew J. Viterbi and James K. Omura, *Principles of Digital Communication and Coding*, McGraw-Hill, Inc., New York, NY, USA, 1979.

[3] Henrik Schulze and Christian Lüders, *Theory and Applications of OFDM and CDMA - Wideband Wireless Communications*, John Wiley & Sons, Ltd, 2005

[4] M.K. Simon and M.S. Alouini, *Digital Communications over Fading Channels*, Wiley, 2000

[5] N.G. van Kampen *Stochastic Processes in Physics and Chemistry*, North-Holland Publishing Company, 1981

[6] Joachim Hagenauer, *Rate-compatible punctured convolutional codes (RCPC codes) and their applications*, IEEE Transactions on Communications, 36(4):389–400, April 1988.